Solutions to JEE Main Home Practice Test - 7 | JEE - 2024

PHYSICS

SECTION-1

1.(C) Least count (LC) $= \dfrac{\text{Pitch}}{\text{Number of divisions on circular scale}} = \dfrac{1}{100} mm$

$= 0.01\ mm$

As zero is not hidden from circular scale when A and B touches each other. Hence, the screw gauge has positive error.

$$e = +n(LC) = 32 \times 0.01 = 0.32\ mm$$

Linear scale reading = 4 × (1 mm) = 4 mm

Circular scale reading = 16 × (0.01 mm) = 0.16 mm

∴ Measured reading = (4 + 0.16) mm = 4.16 mm

∴ Absolute reading = Measured reading – e

= (4.16 – 0.32) mm = 3.84 mm

Therefore, thickness of the glass plate is 3.84 mm

2.(D) The *rms* speed of a gas molecule is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

The *rms* speed of helium at –20°C (253 K) is given by

$$= \sqrt{\frac{3 \times R \times 253}{4 \times 10^{-3}}} \quad \ldots \text{(i)}$$

The *rms* speed of argon at T is given by

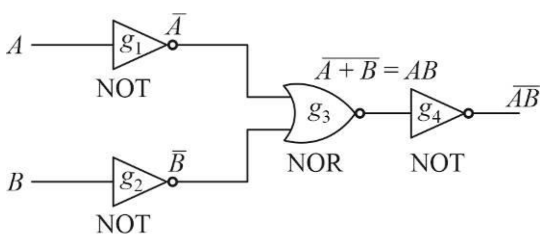
$$= \sqrt{\frac{3 \times R \times T}{39.9 \times 10^{-3}}} \quad \ldots \text{(ii)}$$

Since when the *rms* speed of two gas atoms equals (given) *i.e.*, Eq. (i) = Eq. (ii)

$$\therefore \quad \sqrt{\frac{3 \times R \times 253}{4 \times 10^{-3}}} = \sqrt{\frac{3 \times R \times T}{39.9 \times 10^{-3}}}$$

$$\therefore \quad T = \frac{253 \times 39.9}{4} = 2523.67K = 2.52 \times 10^3 K$$

3.(D)



Hence, this is NAND gate.

4.(C) $\dfrac{\Delta Q}{\Delta t} = \dfrac{\Delta U}{\Delta t} + \dfrac{\Delta W}{\Delta t}$

$$\frac{12000}{60} = \frac{2.5 \times 10^3}{\Delta t} + 100$$

$\Delta t = 25\,\text{sec}$

5.(B) Let ground state energy (in eV) be E_1

The from the given condition

$$E_{2n} - E_1 = 204\,eV \qquad \text{or} \qquad \frac{E_1}{4n^2} - E_1 = 204\,eV$$

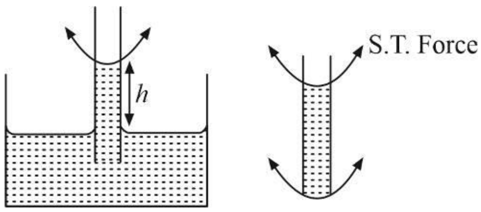
$\Rightarrow \qquad E_1\left(\frac{1}{4n^2} - 1\right) = 204\,eV$

and $\qquad E_{2n} - E_n = 40.8\,eV$

$\Rightarrow \qquad \frac{E_1}{4n^2} - \frac{E_1}{n^2} = E_1\left(-\frac{3}{4n^2}\right) = 40.8\,eV$

From equation (i) and (ii), $\dfrac{1 - \dfrac{1}{4n^2}}{\dfrac{3}{4n^2}} = 5 \Rightarrow n = 2$

6.(A) When the capillary is inside the liquid, the surface tension force supports the weight of liquid of height 'h'



When the capillary is taken out from the liquid similar type of surface tension force acts at the bottom also, as shown in figure. Hence now it can support weight of a liquid of height $2h$.

7.(D) From $E = W_0 + \frac{1}{2}mv_{\max}^2$

$\Rightarrow \qquad 2h\nu_0 = h\nu_0 + \frac{1}{2}mv_1^2 \qquad \Rightarrow \qquad h\nu_0 = \frac{1}{2}mv_1^2$

and $\qquad 5h\nu_0 = h\nu_0 + \frac{1}{2}mv_2^2 \qquad \Rightarrow \qquad 4h\nu_0 = \frac{1}{2}mv_2^2$

Dividing equation (ii) by (i) $\left(\frac{v_2}{v_1}\right)^2 = \frac{4}{1}$

$\Rightarrow \qquad v_2 = 2v_1 = 2 \times 4 \times 10^6 = 8 \times 10^6\,m/s$

8.(C) Let v_1 be the speed of gun (or mirror) just after the firing of bullet. From conservation of linear momentum,

$$m_2 v_0 = m_1 v_1$$

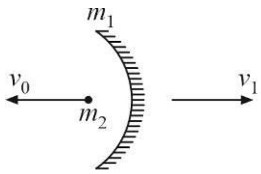
or $\qquad v_1 = \frac{m_2 v_0}{m_1}$

Now, $\frac{du}{dt}$ is the rate at which distance between mirror and bullet is increasing $= v_1 + v_0$

$\therefore \qquad \frac{du}{dt} = \left(\frac{v^2}{u^2}\right)\frac{du}{dt}$

Here, $\frac{v^2}{u^2} = m^2 = 1$ (as at the time of firing, bullet is at pole)

$$\frac{dv}{dt} = \frac{du}{dt} = v_1 + v_0$$



Here, $\frac{du}{dt}$ is the rate at which distance between image (of bullet) and mirror is increasing. So v_2 is the absolute velocity of image (towards right), then,

$$v_2 - v_1 = \frac{dv}{dt} = v_1 + v_0$$

or $\quad v_2 = 2v_1 + v_0$

Therefore, speed of separation of bullet and image will be

$$v_r = v_2 + v_0 = 2v_1 + v_0 + v_0$$

or $\quad v_r = 2(v_1 + v_0)$

Substituting value of v_2 from equation (i), we have

$$v_r = 2\left(1 + \frac{m_2}{m_1}\right)v_0$$

9.(D) $U = \frac{1}{2}Li^2 = 32$

$$\frac{1}{2} \times L \times 8^2 = 32$$

$$L = 1$$

Also, $\quad i^2R = 640$

$$R = 10$$

Time constant $= \frac{L}{R} = \frac{1}{10} = 0.1$

10.(B) $f_R = \frac{1}{2\pi\sqrt{LC}}$

11.(B) Using Biot Savart Law

12.(B) Using symmetry, both 6Ω resistors will be removed.

13.(A) $r < b$

$E_r = 0;\qquad b \le r < c$

$$E_r = \frac{2kq}{r^2}$$

$c \le r < d$

$E_r = 0;\qquad r \ge d$

$$E_r = \frac{6kq}{r^2}$$

14.(B) This is basically a problem of discharging of a capacitor from inside the capacitor. Charge at any time t is

Here, $q_0 =$ (area of plates) (surface charge density)

And discharging current, $i = \left(\frac{-dq}{dt}\right) = \frac{q_0}{\tau_C} \cdot e^{-t/\tau_C} = i_0 e^{-t/\tau_C}$

Here, $i_0 = \frac{q_0}{\tau_C} = \frac{q_0}{CR}$

$C = \frac{K\varepsilon_0 A}{d}$ and $R = \frac{d}{\sigma A}$

$CR = \frac{K\varepsilon_0}{\sigma}$

Therefore, $i_0 = \frac{q_0}{\frac{K\varepsilon_0}{\sigma}} = \frac{\sigma q_0}{K\varepsilon_0} \quad \Rightarrow \quad K = \frac{\sigma q_0}{i_0 \varepsilon_0}$

Substituting the values, we have

$$K = \frac{(5.0\times10^{-14})(\pi)(4.0)^2(15\times10^{-6})}{(1.0\times10^{-6})(8.86\times10^{-12})}$$

$$U = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}\frac{q_0^2}{\frac{K\varepsilon_0 A}{d}}$$

$$d = \frac{2K\varepsilon_0 AU}{q_0^2}$$

$$= \frac{2\times4.25\times8.86\times10^{-12}\times\pi\times(4.0\times10^{-2})^2\times7500}{(15\times10^{-6}\times\pi\times4.0\times4.0)^2}$$

$$= 5.0\times10^{-3} m = 5.0\, mm$$

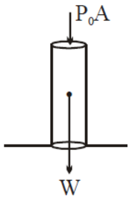
15.(B) $F = 0$ where $0 \le r \le R_1$

Because intensity is zero inside the cavity (using gauss law)

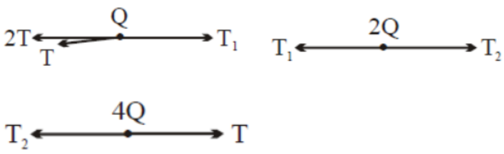
16.(D) $P_0A + hA\rho g = P_1A$

$h = \frac{P_1 - P_0}{\rho g}$

$= \frac{12\times10^5}{4\times10^3\times10} = 30$ m



17.(B)



$T_2 = T + 8T$

$= 9T$

$T_1 = 3T$

18.(D) In ΔMNO

$\vec{A}+\vec{C}-\vec{D}=0$

$\vec{C}-\vec{D}=-\vec{A}$

In ΔMNP

$\vec{A}+\vec{B}+\vec{E}=0$

In MPNO

$-\vec{E}-\vec{B}+\vec{C}-\vec{D}=0$

$\vec{B}+\vec{E}-\vec{C}=-\vec{D}$

19.(A) Theory Based

20.(B) $L \propto v^x A^y F^z \Rightarrow L = kv^x A^y F^z$

Putting the dimensions in the above relation

$$[ML^2T^{-1}] = k[LT^{-1}]^x\,[LT^{-2}]^y\,[MLT^{-2}]^z$$

$$\Rightarrow \quad [ML^2T^{-1}] = k[M^z L^{x+y+z}\,T^{-x-2y-2z}]$$

Comparing the powers of M, L and T

$z = 1$ … (i)

$x + y + z = 2$ … (ii)

$-x - 2y - 2z = -1$ … (iii)

On solving (i), (ii) and (iii) $x = 3$, $y = -2$, $z = 1$

So dimension of L in terms of v, A and F

$[L] = [Fv^3A^{-2}]$

SECTION - 2

1.(0) Let L_1 and L_2 be the portions (of length) of rope on left and right surface of wedge acceleration =

$$\frac{\frac{M}{L}[L_1 \sin\alpha - L_2 \sin\beta]}{M} = 0 \qquad [\because\ L_1 \sin\alpha = L_2 \sin\beta]$$

2.(3) Let m_1 = mass of the square plate of side 'a'

and m_2 = mass of the square of side '$a/2$'

Then $m_1 = \sigma\left(\frac{a}{2}\right)^2$ and $m_2 = \sigma(a)^2$ (σ being the areal density and) $m_2 - m_1 = M$

$$\Rightarrow \quad I = \frac{m_2a^2}{6} - \left\{\frac{m_1(a/2)^2}{6} + m_1\left(\frac{a}{4}\right)^2\right\} = \frac{\sigma a^4}{6} - \left\{\frac{\sigma(a/2)^4}{6} + \sigma\left(\frac{a}{2}\right)^2 . \left(\frac{a}{4}\right)^2\right\}$$

$$\Rightarrow \quad I = \sigma a^4\left\{\frac{27}{12\times16}\right\}$$

Also $\quad M = \sigma\left(1 - \frac{1}{4}\right)a^2 \quad \Rightarrow \quad \sigma = \frac{4}{3}\frac{M}{a^2}$

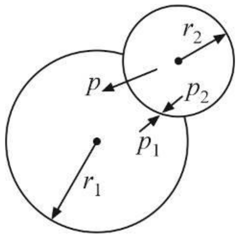
$$\Rightarrow \quad \left(\frac{4}{3}\frac{M}{a^2}\right)\cdot a^4\left\{\frac{27}{12\times16}\right\} \Rightarrow \quad I = \frac{3Ma^2}{16}$$

3.(4) $p_1 = p_0 + \frac{4T}{r_1} \Rightarrow \quad p_2 = p_0 + \frac{4T}{r_2}$

$r_2 < r_1$

$p_2 > p_1$

i.e. pressure inside the smaller bubble will be more. The excess pressure



This excess pressure acts from concave to convex side the interface will be concave towards smaller bubble and convex towards larger bubble. Let R be the radius of interface then,

$$p = \frac{4T}{R}$$

From equation (i) and (ii), we get

$$R = \frac{r_1 r_2}{r_1 - r_2} = \frac{(0.004)(0.002)}{(0.004 - 0.002)} = 0.004\, m$$

4.(4) At $x = 0$ the phase difference should be π

$\therefore$ The correct option is (D)

Alternate Method

$y_2 = a\cos(\omega t + kx + \phi_0)$

$$\therefore \quad y = y_1 + y_2 = a\cos\left(\omega t - kx + \frac{\pi}{3}\right) + a\cos(\omega t + kx + \phi_0)$$

$$= 2a\cos\left[\omega t + \frac{\frac{\pi}{3} + \varphi_0}{2}\right] \times \cos\left[kx + \frac{\varphi_0 - \frac{\pi}{3}}{2}\right]$$

$\therefore \quad y = 0$ at $x = 0$ for any $t \Rightarrow kx + \frac{\varphi_0 - \frac{\pi}{3}}{2} = \frac{\pi}{2}$ at $x = 0 \quad \therefore \quad \phi_0 = \frac{4\pi}{3}$

Hence $y_2 = a\cos\left(\omega t + kx + \frac{4\pi}{3}\right)$

5.(1) $a = \frac{-5\times 10^{-2}}{20\times 10^{-3}}$, $v^2 = u^2 + 2as = 1^2 - \frac{2\times 5\times 10^{-2}}{20\times 10^{-3}} \times 20\times 10^{-2} = 1 \Rightarrow v \simeq 1\, m/s$

6.(5) Here, $I = 1\, kW/m^2 = 10^3\, W/m^2$

Radiation pressure exerted by absorbed light

$$= \frac{1}{2}\left(\frac{I}{c}\right)$$

Radiation pressure exerted by reflected light

$$= \frac{1}{2}\left(\frac{2I}{c}\right) = \frac{I}{c}$$

Total radiation pressure on the surface

$$P_{\text{rad}} = \frac{3I}{2c} = \frac{3\times10^3}{2\times(3\times10^8)} = 5\times10^{-6}\,Pa$$

7.(6) $x = n\lambda\frac{D}{d} = (n+1)\lambda'\frac{D}{d}$

$\therefore$ $n\times12000 = (n+1)10000$ … (i)

2000 n = 10000

n = 5

From (i), $x = \frac{5\times(12000\times10^{-10})\times2}{2\times10^{-3}} = 6\times10^{-3}m = 6.0\,mm$

8.(1) For string, $\frac{\text{Mass}}{\text{Length}} = m = \frac{10^{-2}}{0.4} = 2.5\times10^{-2}\,kg/m$

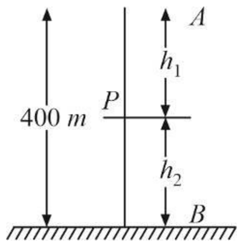
$\therefore$ Velocity $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{16}{2.5\times10^{-2}}} = 8m/s$

For constructive interference between successive pulses.

$$\Delta t_{\min} = \frac{2l}{v} = \frac{2(4)}{8} = 1\text{sec}$$

(After two reflections, the wave pulse is in same phase as it was produced since in one reflection it's phase changes by π, and If at this moment next identical pulse is produced, then constructive interference will be obtained.

9.(8) Let both balls meet at point P after time t.



The distance travelled by ball A, $h_1 = \frac{1}{2}gt^2$

The distance travelled by ball B, $h_2 = ut - \frac{1}{2}gt^2$

$$h_1 + h_2 = 400\,m$$

$\Rightarrow$ $ut = 400, t = \frac{400}{50} = 8\text{sec}$

$\therefore$ $h_1 = 320$ and $h_2 = 80\,m$

10.(5) $mur = m(v_0 + v)a$

$\Rightarrow u = \sqrt{\frac{5GM_e a}{4r^2}}$ as $v_0 = \sqrt{\frac{GM_e}{a}}$

$$\frac{1}{2}mu^2 - \frac{GM_e m}{r} = \frac{1}{2}m(v_0+v)^2 - \frac{GM_e m}{a}$$

CHEMISTRY

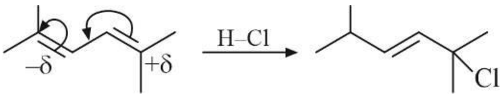
SECTION-1

1.(D) Interhalogens compound are more reactive than halogens [exception: F_2 due to repulsion force between F – F atoms].

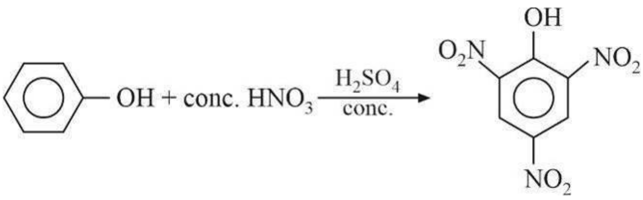
$$XX' + H_2O \rightarrow HX + HOX'$$

Bigger X' forms hypohalite

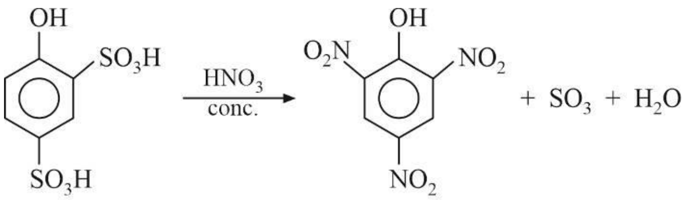
Smaller X forms X^-

2.(A)  (1, 4 addition)

3.(B) Nitration:



(Poor yield, because NO_2 group is deactivating)



(Picric acid)

4.(D) $Z\downarrow \quad r\uparrow$

$n\downarrow \quad r\downarrow$

$$r = \frac{n^2h^2}{4\pi^2 k\, m_e\, e^2\, Z}$$

5.(A) For adsorption, $\Delta H > 0$ is not possible.

6.(B)

$$\underset{(X)}{CaCO_3} \xrightarrow{\Delta} CO_2 + CaO$$

$$CaO + H_2O \longrightarrow \underset{(Y)}{Ca(OH)_2}$$

$$Ca(OH)_2 + \underset{(excess)}{2CO_2} \longrightarrow \underset{(Z)}{Ca(HCO_3)_2}$$

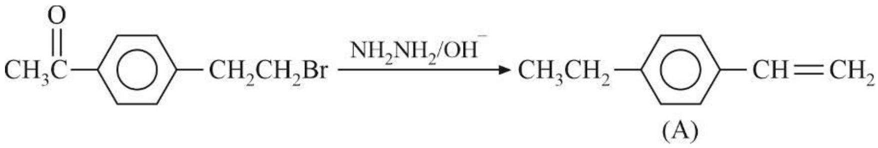
$$\underset{(Z)}{Ca(HCO_3)_2} \xrightarrow{\Delta} \underset{(X)}{CaCO_3} + CO_2 + H_2O$$

7.(B) CuF_2 : Cu^{2+}, unpaired electron is present therefore coloured

8.(B)

$$\underset{\text{2-Methylbutane}}{CH_3-\overset{\overset{\large CH_3}{|}}{CH}-CH_2-CH_3} \xrightarrow{Br_2/hv} CH_3-\overset{\overset{\large CH_3}{|}}{\underset{\underset{\large Br}{|}}{C}}-CH_2-CH_3$$

9.(B) NH_2NH_2, OH^- will not only reduces $\overset{\overset{\displaystyle O}{\|}}{CH_3C}-$ to CH_3CH_2- but will also bring out dehydrobromination of CH_2CH_2Br to $CH=CH_2$. Thus, product (A) is.

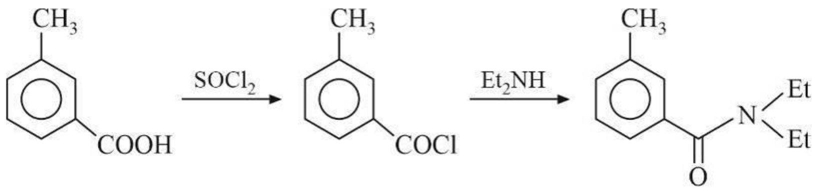


10.(B)

11.(C) For appearing violet complex must have absorbed low energy light.

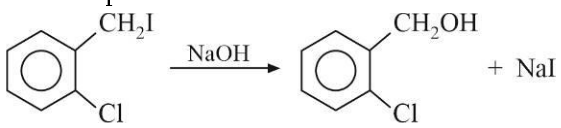
$[CoCl(NH_3)_5]^{2+}$	Violet
$[Co(NH_3)_6]^{3+}$	Yellow Orange
$[Co(CN)_6]^{3-}$	Pale Yellow
$[Cu(H_2O)_4]^{2+}$	Blue

12.(D) More the reduction potential – Strongest oxidising agent.

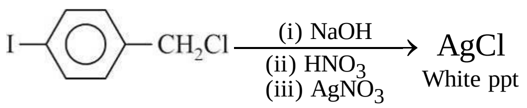
13.(D) 

14.(B) $3Ca_3(PO_4)_2 \cdot CaF_2$ is the compound fluoroapatite.

15.(C) Only benzyl halides, i.e., p-iodobenzyl chloride and o-chlorobenzyl iodide undergo hydrolysis on shaking with an aqueous solution of NaOH. Since yellow ppt. of AgI are obtained, therefore, iodine must be present in the side chain and not in the nucleus. Therefore, option (C) is correct.



$$NaI \xrightarrow[\text{(ii) } AgNO_3]{\text{(i) } HNO_3} \underset{\text{Yellow ppt}}{AgI}$$



16.(D) Phenolphthalein $\Rightarrow$ used for titration of weak acid with strong base.
Methyl orange $\Rightarrow$ used for titration of weak base with strong acid.

17.(C) $k_2 = Ae^{\frac{\varepsilon_{a_2}}{RT}}$

$k_1 = Ae^{\frac{-\varepsilon_{a_1}}{RT}}$

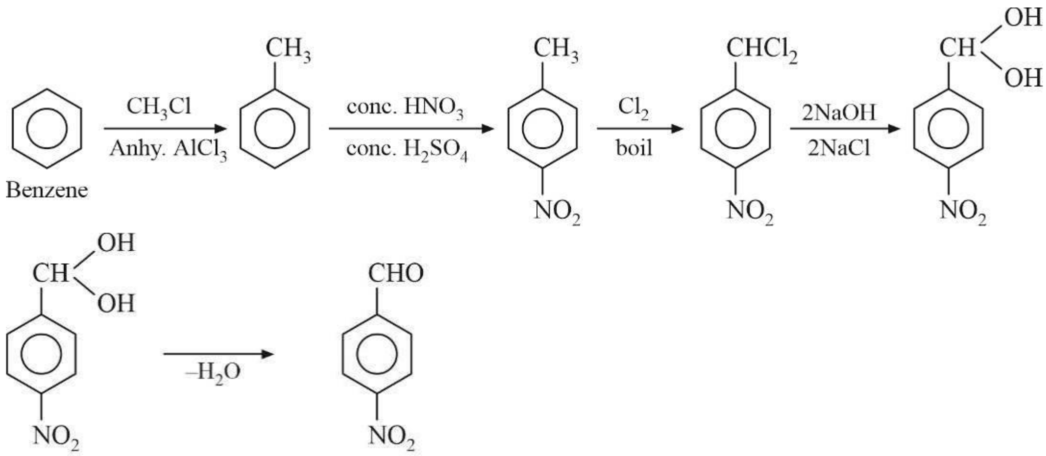
$= e^{\frac{-\left(\varepsilon_{a_2} - \varepsilon_{a_1}\right)}{RT}}$

$$\frac{k_2}{k_1} = \frac{Ae^{-\varepsilon_{a_2}/RT}}{Ae^{-\varepsilon_{a_1}/RT}}$$

$$k_2 = k_1 e^{-\varepsilon_q/RT}$$

18.(C) Molar conductivity of only strong electrolytes varies linearly on dilution.

19.(C)



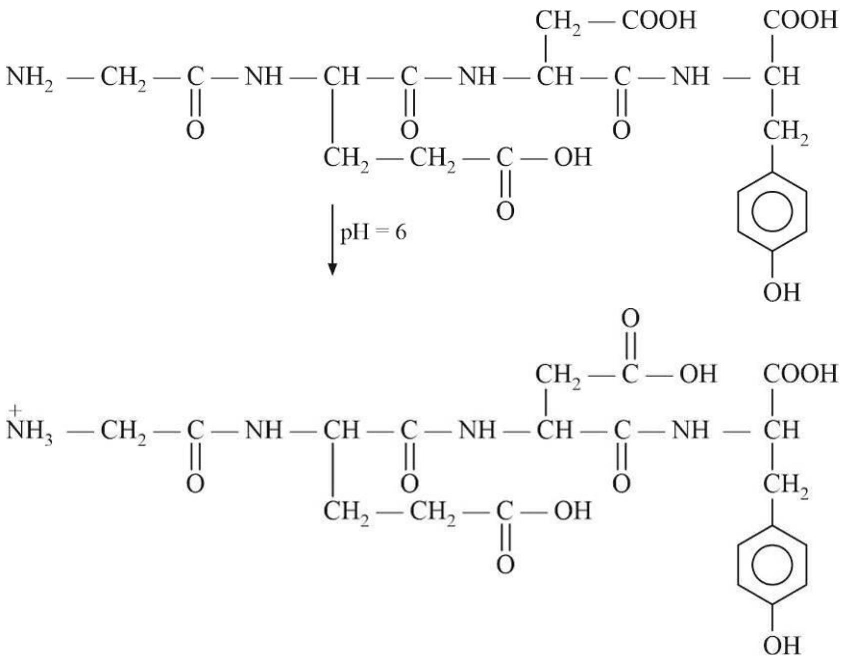
4-nitro benzaldehyde

20.(B)

SECTION - 2

1.(3) (ii), (iii) and (iv) as 'i' is less than i of $Al_2(SO_4)_3$.

2.(1)



3.(4) $K_b = \dfrac{[(CH_3)_3NH^+][OH^-]}{[(CH_3)_3N]}$

$$\Rightarrow \quad [OH^-] = \frac{K_b[(CH_3)_3N]}{[(CH_3)_3NH^+]} = \frac{6.3\times10^{-5}\times0.35}{0.05} = 4.41\times10^{-4}\,M$$

4.(788)

By Born-Haber cycle,

$$\Delta_f H^\circ (\text{NaCl, s}) = \Delta_{\text{sub}}H^\circ (\text{Na}) + \frac{1}{2}\Delta_{\text{disso}}H^\circ (\text{Cl}_2)$$
$$+ \Delta_{\text{ionisation}}H^\circ (\text{Na}) + \Delta_{\text{eg}}H^\circ (\text{Cl}) + \Delta_{\text{lattice}}H^\circ (\text{NaCl})$$

$$-410 = +109 + \frac{1}{2}(244) + 496 - 349 + \Delta_{\text{lattice}}H^\circ (\text{NaCl})$$

or $\quad \Delta_{\text{lattice}}H^\circ (\text{NaCl}) = -788 \text{ kJ}$

This is energy released when gaseous Na^+ and Cl^- ions combine to form one mole of NaCl (s). Hence, energy required for dissociation of one mole of NaCl (s) into gaseous ions $= +788 \text{ kJ mol}^{-1}$.

5.(7) Magnesium sulphate $MgSO_4 \cdot 7H_2O$ is Epsom salt

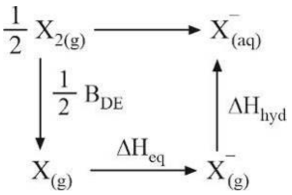
Glauber's salt is $Na_2SO_4 \cdot 10H_2O$

$$\frac{7}{10} = 0.7 = 7 \times 10^{-1}$$

6.(20) ${}_{64}Gd^{3+} : 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 5s^2\ 4d^{10}\ 5p^6\ 4f^7$

7.(3) In ICl_2^-, I has 3 three lone pairs

8.(3)



C, D, E are correct answer

9.(1) Let the density of the solution is $d \text{ g mL}^{-1}$, then 1L, i.e. 1000 mL of solution has mass = 1000 d g

Thus, 1000 d g of the solution contain $H_2SO_4 = 11.5$ moles $= 11.5 \times 98 \text{ g} = 1127 \text{ g}$

$\therefore$ Mass of water = (1000 d – 1127) g

Molality $= \dfrac{11.5}{1000d - 1127} \times 1000 = 94.5$ (Given)

or 11500 = 94.5 × 1000 d – 94.5 × 1127

or 94500 d = 11500 + 106501.5 = 118001.5

$\therefore \quad d = \dfrac{118001.5}{94500} = 1.2487$ g/mL

10.(1) $\dfrac{R_1}{R_2} = \dfrac{1.16}{1.70} = \left(\dfrac{0.15}{0.22}\right)^\alpha ; \Rightarrow \quad \alpha = 1$ (order w.r.t., $S_2O_8^{2-}$)

MATHEMATICS

SECTION-1

1.(D) Consider the product

$$PP^T = \begin{bmatrix} \frac{\sqrt{3}+1}{2\sqrt{2}} & \frac{\sqrt{3}-1}{2\sqrt{2}} \\ \frac{1-\sqrt{3}}{2\sqrt{2}} & \frac{\sqrt{3}+1}{2\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}+1}{2\sqrt{2}} & \frac{1-\sqrt{3}}{2\sqrt{2}} \\ \frac{\sqrt{3}-1}{2\sqrt{2}} & \frac{\sqrt{3}+1}{2\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \qquad \ldots\text{(i)}$$

Now, $Q = P^T AP$

$\Rightarrow \quad Q^2 = (P^T AP)\,(P^T AP) = P^T A^2 P \qquad (\because PP^T = I)$

$\Rightarrow \quad Q^3 = (P^T A^2 P)\,(P^T AP) = P^T A^3 P$

$\Rightarrow \quad Q^{2010} = P^T A^{2010} P$

$\Rightarrow \quad P \cdot Q^{2010} P^T = P \cdot P^T A^{2010} P \cdot P^T = A^{2010}$

$\Rightarrow \quad [\because\ PP^T = I] \quad \Rightarrow \quad \begin{bmatrix} i^{2010} & 0 \\ 0 & i^{2010} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

2.(A) The equation of normal to the parabola $y^2 = 8x$ at $Q(2m^2, -4m)$ is

$$y = mx - 4m - 2m^3 \qquad \ldots\text{(i)}$$

Now, for the minimum distance, the equation of normal [Eq. (i)] is common for both. Thus, the normal [Eq.(i)] must pass through the centre $C(0, -6)$

$$m^3 + 2m - 3 = 0$$

$\Rightarrow \quad (m-1)(m^2 + m + 3) = 0 \qquad \Rightarrow \qquad m = 1$

Thus, the required point is (2, –4)

3.(C) $f = a_1 a_4 a_5$

$a_6 = 2$

$a + 5d = 2$

$= a(a+3d)(a+4d) = (2-5d)(2-5d+3d)(2-5d+4d)$

$= (2-5d)(2-2d)(2-d) = 2(2-5d)(2-d-2d+d^2)$

$= 2(2-5d)(d^2-3d+2) = 2(2d^2-6d+4-5d^3+15d^2-10d)$

$= 2(-5d^3+17d^2-16d+4) = -2[5d^3-17d^2+16d-4]$

$f' = -2[15d^2-34d+16]$

$15d^2 - 34d + 16 = 0$

$15d^2 - 24d - 10d + 16 = 0$

$3d(5d-8)(3d-2) = 0$

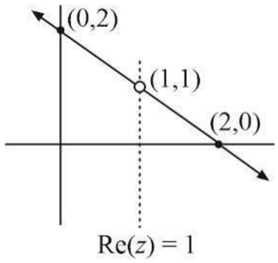
$d = \frac{8}{5}, \frac{2}{3}$

$$f'' = -2[30d - 34] = -2\left[\overset{6}{\cancel{30}} \times \frac{8}{\cancel{5}} - 34\right] = -28$$

At $d = \dfrac{8}{5}$ given function is maximum.

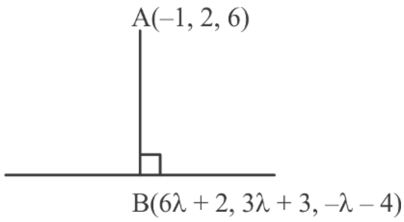
4.(D) See figure, the given equation is written as

$$\arg[z - (1+i)] = \begin{cases} \dfrac{3\pi}{4}, \text{ when } x \le 1 \\ \dfrac{-\pi}{4}, \text{ when } x > 1 \end{cases}$$



Therefore, the locus is a set of two rays.

5.(D) Eqn. of line : $\dfrac{x-2}{6} = \dfrac{y-3}{3} = \dfrac{z+4}{-4} = \lambda$



DR's of AB $\langle 6\lambda + 3, 3\lambda + 1, -4\lambda - 10\rangle$

Given line $\perp$ AB

$\therefore \quad 6(6\lambda+3) + 3(3\lambda+1) - 4(-4\lambda-10) = 0$

$\Rightarrow \quad \lambda = -1$

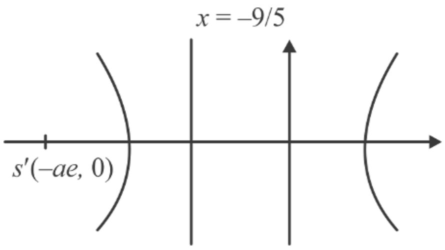
$\therefore \quad$ B(–4, 0, 0)

A(–1, 2, 6)

AB = 7.

6.(A) Obviously the given point lies on the plane 12x – 3y + 4z = 0, whose minimum distance from Q is 1 unit

7.(C) $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$



$b^2 = a^2(e^2 - 1)$

$16 = 9(e^2 - 1) \Rightarrow e = \dfrac{5}{3} \ \ldots(1)$

$-\dfrac{a}{e} = \dfrac{-9}{5} \Rightarrow \dfrac{a}{e} = \dfrac{9}{5}$

$\therefore \quad (-ae, 0) = (-5, 0)$

8.(A) Since $\overline{E}_1 \cap \overline{E}_2 = \overline{E_1 \cup E_2}$ and $(E_1 \cup E_2) \cap (\overline{E_1 \cup E_2}) = \phi$

$$P\{(E_1 \cup E_2) \cap (\overline{E}_1 \cap \overline{E}_2)\} = P(\phi) = 0 < \frac{1}{4}$$

9.(A) $\overline{x} = 7, N = 6, \sum \dfrac{(x-\overline{x})^2}{N} = \dfrac{25}{3}$ and x_1 and x_2 are other two observations, then

$$x_1 + x_2 = 42 - 5 - 6 - 8 - 9 = 14 \qquad \ldots \text{(i)}$$

$$\sum (x-\bar{x})^2 = 50$$

$\Rightarrow \quad (5-7)^2+(6-7)^2+(8-7)^2+(9-7)^2+(x_1-7)^2+(x_2-7)^2=50$

$\Rightarrow \quad (x_1-7)^2+(x_2-7)^2=40$

$\Rightarrow \quad (x_1-7)^2+(14-x_1-7)^2=40$ [From Eq. (i)]

$\Rightarrow \quad (x_1-7)^2+(7-x_1)^2=40$

$\Rightarrow \quad (x_1-7)^2=20 \quad \Rightarrow \quad x_1-7=\pm\sqrt{20} \quad \Rightarrow \quad x_1=7\pm\sqrt{20}$

$\Rightarrow \quad x_1=7+\sqrt{20}$

$\Rightarrow \quad x_2=7-\sqrt{20}$

Now all the 6 observations arranged in ascending order are $7-\sqrt{20}, 5, 6, 8, 9, 7+\sqrt{20}$

Therefore,

$$\text{Median} = \frac{6+8}{2}=7$$

10.(C) $2x^3y\,dy+(1-y^2)(x^2y^2+y^2-1)dx=0$

$\Rightarrow \quad \dfrac{2y}{(1-y^2)^2}\dfrac{dy}{dx}+\dfrac{y^2}{1-y^2}\dfrac{1}{x}=\dfrac{1}{x^3}$

Put $\dfrac{y^2}{1-y^2}=u.$

Then, $\dfrac{2y}{(1-y^2)^2}\dfrac{dy}{dx}=\dfrac{du}{dx}$

$\Rightarrow \quad \dfrac{du}{dx}+\dfrac{u}{x}=\dfrac{1}{x^3} \quad \Rightarrow \quad$ Integrating factor $=e^{\int \frac{1}{x}dx}=e^{\ln x}=x$

Solution is $u\cdot x=\displaystyle\int \frac{1}{x^2}\,dx+c$

$\Rightarrow \quad x^2y^2=(cx-1)\,(1-y^2)$

11.(A) Putting $x=\tan\theta,$ we get

$$\int_0^\infty \frac{dx}{\left(x+\sqrt{x^2+1}\right)^3}=\int_0^{\pi/2}\frac{\sec^2\theta\,d\theta}{(\tan\theta+\sec\theta)^3}=\int_0^{\pi/2}\frac{\cos\theta}{(1+\sin\theta)^3}\,d\theta=\left[-\frac{1}{2(1+\sin\theta)^2}\right]_0^{\pi/2}$$

$$=-\frac{1}{8}+\frac{1}{2}=\frac{3}{8}$$

12.(A) As the given system of equations has a non-trivial solution,

$$\begin{vmatrix} \lambda & b-a & c-a \\ a-b & \lambda & c-b \\ a-c & b-c & \lambda \end{vmatrix}=0$$

When $\lambda=0$ then determinants become skew-symmetric determinants of odd order, which is equal to zero. Thus, $\lambda=0.$

13.(A) For the increasing function $f'(x) > 0$

$$x^3 + 2x^2 + 5x = -2\cos x \quad \ldots \text{(i)}$$

Let $f(x) = x^3 + 2x^2 + 5x \quad \because \quad f'(x) = 3x^2 + 4x + 5$

Also, $f'(x) > 0$ and so $f(x)$ is always increasing

Curves in equation (i) have no intersection points.

14.(C) $\lim\limits_{x \to 1} \dfrac{x^2 - ax + b}{x - 1} = 5$

$1 - a.1 + b = 0 \quad \ldots(1)$

$\lim\limits_{x \to 1} \dfrac{2x - a}{1} = 5$

$2 - a = 5 \Rightarrow a = -3 \quad \ldots(2)$

From (1) and (2) $b = -4$

$\therefore \quad a + b = -7.$

15.(C) Common ratio = 2, a = 1

$x_1 = at_1^2 = 1 \Rightarrow t_1 = \sqrt{1}$

$x_2 = x_1 \cdot r = 2 \Rightarrow t_2 = \sqrt{2}$

$x_3 = x_1 \cdot r^4 = 4 \Rightarrow t_3 = \sqrt{4}$

$\vdots$

$y_1 = 2at_1 = 2\sqrt{1}$

$y_2 = 2at_2 = 2\sqrt{2}$

$y_3 = 2at_3 = 2\sqrt{4}$

$\vdots$

$y_n = 2at_n = 2\sqrt{2^{n-1}} = 2.2^{\frac{n-1}{2}} = 2^{\frac{n+1}{2}}$

16.(C) $1 - P(x = 0) = 1 - {}^nC_0 \left(\dfrac{1}{2}\right)^n > \dfrac{99}{100}$

$\therefore \quad \dfrac{1}{100} > \dfrac{1}{2^n}$

$2^n > 100 \quad \therefore$ Least n = 7.

17.(B) $\cos\left(\tan^{-1}\sqrt{\dfrac{1-x}{x}}\right) = \sqrt{x}$

$f(x) = \sin(2\tan^{-1}\sqrt{x})$, Let $\tan^{-1}\sqrt{x} = \theta$

$f(x) = \dfrac{2\tan\theta}{1 + \tan^2\theta} = \dfrac{2\sqrt{x}}{1 + x}$

18.(B) $P(A \cap B) = P(A) \cdot P(B) \quad \Rightarrow \quad 3K^2 = \dfrac{1}{3} \quad \Rightarrow \quad K = \dfrac{1}{3}$

P(exactly one of A, B occurs) $= P(A) + P(B) - 2P(A \cap B)$

19.(A) $S = \sum_{r=1}^{30} r^2 \times {}^{30}C_r = 30 \sum_{r=1}^{30} r\ {}^{29}C_{r-1}$

$$S = 30\left[\sum_{r=1}^{30} (r-1)\ {}^{29}C_{r-1} + \sum_{r=1}^{30} {}^{29}C_{r-1}\right]$$

$$S = 30\left[\sum_{r=1}^{30} 29 \times {}^{28}C_{r-2} + 2^{29}\right]$$

$$S = 30\,[29 \times 2^{28} + 2^{29}]$$

20.(C) $\vec{a}\cdot(\vec{b}\times\vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{b}\ \vec{a}\ \vec{c}] = -\vec{b}\cdot(\vec{a}\times\vec{c}) = -\vec{b}\cdot\vec{b} = -|\vec{b}|^2$

SECTION - 2

1.(7) Let moving point be $P(h, k)$

$\Rightarrow \quad (h-1)^2 + (k-1)^2 + (h-1)^2 + (k+1)^2 + (h+1)^2 + (k-1)^2 + (h+1)^2 + (k+1)^2 = 36$

$\Rightarrow \quad 4[h^2 + k^2 + 2] = 36 \quad \Rightarrow \quad h^2 + k^2 = 7 \quad \Rightarrow \quad x^2 + y^2 = 7$

2.(38) $T_{r+1} = {}^{n}C_r (x^2)^{n-r}\left(\frac{1}{x^3}\right)^r = {}^{n}C_r x^{2n-5r}$

$\therefore \quad 2n - 5r = 1; \quad r = \frac{2n-1}{5}$

$\frac{2n-1}{5} = 23 \Rightarrow n = 58$

$n - \left(\frac{2n-1}{5}\right) = 23 \Rightarrow n = 38 \quad \therefore \quad n = 38.$

3.(4) Let $S = \left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \ldots + \left(z^{27} + \frac{1}{z^{27}}\right)^2$

$$S = z^2 + z^4 + z^6 + \ldots + z^{54} + \left(\frac{1}{z^2} + \frac{1}{z^4} + \ldots + \frac{1}{z^{54}}\right) + 54$$

$$S = \frac{z^2(1 - z^{54})}{1 - z^2} + \frac{\frac{1}{z^2}\left(1 - \frac{1}{z^{54}}\right)}{1 - \frac{1}{z^2}} + 54$$

$z = -\omega$ (where, 'ω' is an imaginary cube root of unity)

$S = 54$

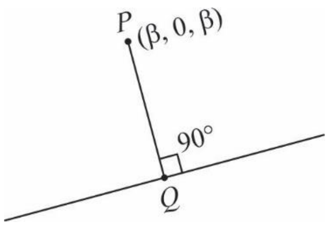
4.(4) Given line is $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ and the point is $(\beta, 0, \beta)$ where $\beta \neq 0$.

Let coordinate of $Q(\lambda, 1, -\lambda - 1)$

$(\lambda - \beta)\cdot 1 + (1 - 0)\cdot 0 + (-\lambda - 1 - \beta)(-1) = 0$

From here $\lambda = -\frac{1}{2}$

So, the point Q becomes



$\left(-\frac{1}{2}, 1, -\frac{1}{2}\right)$

Now $\sqrt{\left(\beta+\frac{1}{2}\right)^2+1+\left(\beta+\frac{1}{2}\right)^2} = \sqrt{\frac{3}{2}}$

So, $\beta = -1$

5.(5) $\frac{2}{x-1}-\frac{1}{x-2}=\frac{1}{\lambda}$

$\Rightarrow \quad x^2-(3+\lambda)x+2+3\lambda=0$

$D<0$

$\Rightarrow \quad (3+\lambda)^2-4(2+3\lambda)<0 \quad \Rightarrow \quad \lambda \in (3-2\sqrt{2}, 3+2\sqrt{2})$

6.(4) $2x+y=e^{2xy} \quad \Rightarrow \quad$ If $x=0, y=1$

$\Rightarrow \quad 2+\frac{dy}{dx}=e^{2xy}\times 2\left(y+x\frac{dy}{dx}\right) \quad \Rightarrow \quad$ If x = 0, $\frac{dy}{dx}=0$

$\Rightarrow \quad (2+y')=2(2x+y)(y+xy') \quad \Rightarrow \quad y''=2\,[(2+y')(y+xy')+(2x+y)(y'+y'+xy'')]$

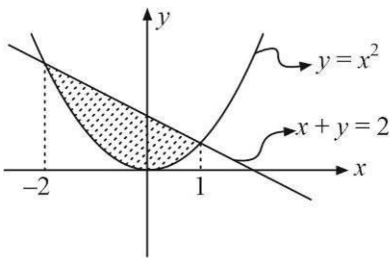
Put $\quad x=0,\ y=1,\ y'=0$

$y''=4$

7.(3)

$$\left.\begin{array}{l}\lim\limits_{x\to 0^+} a\sin\frac{\pi}{3}\left(5x-\frac{3}{2}\right)=-a,\ f(0)=-a \\ \lim\limits_{x\to 0^-} \frac{3}{b}\left(\frac{\tan 3x}{3x}-\frac{2\sin 3x}{3x}\right)=-\frac{3}{b}\end{array}\right]\Rightarrow ab=3$$

8.(9)



Area $=\int_{-2}^{1}(2-x-x^2)\,dx$

9.(2) Let circumference of circle be 'x' and perimeter of square is "$60-x$".

Sum of areas $\quad (S)=\pi R^2+a^2$

$$S=\pi\left(\frac{x}{2\pi}\right)^2+\left(15-\frac{x}{4}\right)^2$$

'S' is minimum at $x=\frac{60\pi}{4+\pi}$

10.(5)

$\underline{5\text{ ways}}\ \underline{5\text{ ways}}\ \underline{3\text{ ways}}$

$\uparrow$ = 75 ways

5, 7, 9